Acoustic phonons mediated non-equilibrium spin current in the presence of Rashba and Dreesselhaus spin-orbit couplings

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Abstract

The influence of the electrons interaction with longitudinal acoustic phonons on magnetoeletric and spin-related transport effects are investigated. The physical system under consideration has been assumed to be a two dimensional electron gas system with both Rashba and Dreesselhaus spin-orbit couplings. In the works which have been previously performed in this field it has been shown that in the non-equilibrium regime, the Rashba and Dreesselhaus couplings can not be responsible for spin-current where in the absence of other interactions such as lattice vibrations identically vanishes. In the current work we have employed a semi-classical method by using the Boltzmann approach. It was shown that the spin-current of the system, in general, does not go all the way to zero at any value of the spin-orbit couplings. It was also shown that the spin accumulation of the system can be influenced by the electron-phonon coupling.

Keywords: spintronics- spin polarized transport- electron-phonon coupling- Rashba and Dreesselhaus spin-orbit couplings- non-equilibrium

spin current.

1. Introduction

Spintronics, has been subject to many investigations and has attracted more and more attention, from both theoretical and experimental sides, during the last several years. Effective control of spin polarized transport is very important specially for practical applications for example in multilayers and many studies have been conducted to explain this phenomena[1, 2].
According to the results of these studies, electron spin manipulation can be realized by applying magnetic fields or the Rashba interaction that arises from the inversion asymmetry in the system and can be effectively controlled by applying a gate voltage [3, 4, 5]. The most popular method for manipulation of electron spin is applying the Rashba spin-orbit interaction (SOI). This kind of spin-orbit interaction plays a central role in Datta and Das spin field-effect transistor (SFET)[6]. Generally SOI has a significant role in magnetoresistance effects known as weak localization [7]. There are many interesting features that have been demonstrated for this type of spin-orbit coupling (SOC) in the field of spin-transport[8, 9, 10]. For example it has been verified that spin-orbit scattering can induce localization/antilocalization transition in a two-dimensional electron gas (2DEG) system[11, 12]. Meanwhile Rashba interaction has been also suggested for spin interference devices and spin-filters [13, 14, 15].
The other spin-orbit coupling that provides us a new spin-dependent parameter which should be considered to design spin-dependent devices, is the Dreesselhaus coupling. As it is known, Dreesselhaus coupling is induced by

the bulk inversion asymmetry. The effects of spin-orbit couplings (SOCs) in semiconductors have attracted growing interest due to their rules in semiconductor spintronics.

Manipulation of spin makes new functionality in electronic devices. Control of spin accumulation by spin-orbit interactions has a great potential in the field of spintronics [16, 17, 18, 19]. In the presence of these two different spin-orbit interactions, i.e. the Rashba and Dreesselhaus couplings, in a two dimensional electron gas system, one can effectively control both magnitude and direction of non-equilibrium spin accumulation[18]. Meanwhile spin current vanishes exactly, which implies that no spin-polarized current accompany with the spin accumulation of the system in non-equilibrium regime induced by an in-plane driving electric field [18].

Rashba obtained non-vanishing spin-current in equilibrium state, therefore this spin-current can not describe any real transport of spins in non-equilibrium regime induced by an in-plane driving electric field [20]. Meanwhile for a non-equilibrium system highly anisotropic spin response to an in-plane electric field has been discovered [21]. However, as mentioned before based on the semiclassical approach, Huang and Hu have shown that the non-equilibrium spin current vanishes exactly in two-dimensional electron gases, in the presence of both Rashba and Dresselhaus couplings, although one can obtain non-equilibrium spin accumulation in this case [18]. In addition Inoue and et al obtained similar results based on the Green's function approach for two-dimensional electron gases [22].

It should be noted that the work describe by Huang and Hu [18], has been based on a semi-classical Boltzmann approach developed by Schliemann and

Loss in [23]. They formulated the anisotropic effects of the energy dispersion relation and scattering matrixes in the presence of spin-orbit couplings. Meanwhile the exact solution to the Boltzmann equation for two-dimensional anisotropic systems has been performed by Výborný and et al [24], where it was shown that for a Rashba type two-band model the discrepancy between the exact and approximative Schliemann and Loss approach, remains only on the level of the higher order corrections [24]. Therefore we have employed the Schliemann and Loss approach so that the results of the current work can be comparable with the results of the Huang and Hu [18] in the same theoretical framework. It can be easily shown that in the presence of both of the spin-orbit couplings i.e. Rashba and Dresselhaus interactions, Schliemann and Loss method is still a good approximation in comparison with the exact Výborný approach. This is due to the fact that the anisotropic term of the energy dispersion relation is negligible and the anisotropic effects can be inter onely through the scattering matrixes.

In this work, based on the mentioned semiclassical approach, we have considered the influence of electron-phonon scattering on spin-transport characteristics of a two-dimensional electron gas with forgoing spin-orbit interactions. We have verified that the electron-phonon scattering results in non-vanishing spin-current that can be controlled by spin-orbit interactions. It has also been found that the spin-current is influenced by the electron-phonon coupling. The details of the numerical results have been briefly addressed in the present work.

2. Model and approach

The total Hamiltonian is given by

$$\hat{H} = \hat{H}_0 + \hat{V}_{im} + \hat{H}_{el-ph}, \tag{1}$$

in the above Hamiltonian, $\hat{H}_0$ is the kinetic energy and spin-orbit interactions (including both Rashba and Dresselhaus spin-orbit couplings), for a 2DEG namely

$$\hat{H}_0 = \frac{\hbar^2 k^2}{2m} + \alpha(\hat{\sigma}_x k_y - \hat{\sigma}_y k_x) + \beta(\hat{\sigma}_x k_x - \hat{\sigma}_y k_y), \tag{2}$$

where $\mathbf{k}$ is the wave vector of conduction electrons, $\sigma_i (i = x, y)$ are Pauli matrices and α and β denote the Rashba and Dresselhaus strengths, respectively.

For a given wave vector $\mathbf{k}$,

$$\mid \mathbf{k}\lambda >= \frac{1}{\sqrt{2}} \begin{pmatrix} e^{i\frac{\phi_{\mathbf{k}}}{2}} \\ \lambda e^{-i\frac{\phi_{\mathbf{k}}}{2}} \end{pmatrix}, \tag{3}$$

are the eigenfunctions of $\hat{H}_0$ where $\lambda = \pm 1$ and $\phi_{\mathbf{k}}$ is defined easily by

$$\tan \phi_{\mathbf{k}} = \frac{\alpha k_x + \beta k_y}{\alpha k_y + \beta k_x}. \tag{4}$$

The corresponding eigenvalues of $\hat{H}_0$ are

$$\epsilon_{\mathbf{k}\lambda} = \frac{\hbar^2 k^2}{2m} + \lambda\sqrt{(\alpha^2 + \beta^2)k^2 + 4\alpha\beta k_x k_y}. \tag{5}$$

The expectation values of the electrons spins along to the x and y directions in the state $\mid \mathbf{k}\lambda >$ cab be easily find to be

$$S^{(0)}_{\lambda,x}(\mathbf{k}) = \frac{\hbar}{2}\lambda\cos(\phi_{\mathbf{k}}), \quad S^{(0)}_{\lambda,y}(\mathbf{k}) = \frac{-\hbar}{2}\lambda\sin(\phi_{\mathbf{k}}). \tag{6}$$

The second term of Hamiltonian, $V_{im}(r)$ is the impurity scattering potential, and can be defined by

$$V_{im}(r) = \sum_i (J\hat{\sigma}.\vec{m}(\mathbf{r}))\delta(\mathbf{r} - \mathbf{r_i}), \tag{7}$$

where the sum is performed over all of the randomly distributed impurities, J is the exchange interaction strength of magnetic impurities with conduction electrons and $\hat{m}(\mathbf{r})$ is the unit vector along the local magnetization.

$$< \mathbf{k}'\lambda' n_q' \mid V_{im}(r) \mid \mathbf{k}\lambda n_q >= C_{\mathbf{k}',\mathbf{k}}\delta_{\acute{n}_q,n_q} \begin{pmatrix} +J_z & J_x - iJ_y \\ J_x + iJ_y & -J_z \end{pmatrix}. \tag{8}$$

In which

$$J_z = m_z J, \quad J_y = m_y J, \quad J_x = m_x J, \tag{9}$$

and

$$C_{\mathbf{k}',\mathbf{k}} = (1/\sqrt{L_x L_y}) \sum_j \exp(i(\vec{k'} - \vec{k}).\vec{r}_j). \tag{10}$$

For long range magnetic interactions, because of the shape anisotropy, we take $m_z = 0$ and for randomly oriented magnetic moments of impurities, one can assume

$$< m_x >=< m_y >= 0, \quad < m_x^2 >=< m_y^2 >= \frac{1}{2}. \tag{11}$$

Since it was assumed that the magnetic moments of the impurities have been randomly oriented therefore they can not be responsible for any spin-polarized effect such as spin current.

The last term of Hamiltonian, $\hat{H}_{el-ph}$ is the electron-phonon interaction and can be expressed as [25],

$$\hat{H}_{el-ph} = D_{ac}\nabla.\vec{u}(r), \tag{12}$$

here, D_{ac} is defined as deformation potential for electron scattering by acoustic phonons and $\vec{u}(r)$ is the small displacement vector of an ion from its equilibrium position, $\vec{R}$.

for the two-dimensional system, the displacement is determined as

$$\vec{u}(r) = \sum_q \sqrt{\frac{\hbar}{2MN\omega_q}} \hat{e_q}[a_q e^{i\vec{q}.\vec{r}} + a_q^\dagger e^{-i\vec{q}.\vec{r}}], \tag{13}$$

where, M and N are mass and number of ions, respectively. $\hat{e_q}$ is the unit vector in displacement direction and $\omega_q = V_s q$ in which V_s is the sound velocity and q is the wave vector of the longitudinal acoustic phonons.

Using from eq. (**??**), the electron-phonon interaction can be written as

$$\hat{H}_{el-ph} = D_{ac} \sum_q \sqrt{\frac{\hbar}{2MNW_q}} (i\hat{e_q}.\vec{q})[a_q e^{i\vec{q}.\vec{r}} - a_q^\dagger e^{-i\vec{q}.\vec{r}}]. \tag{14}$$

by Defining

$$c(q) = D_{ac} \sqrt{\frac{\hbar}{2MNW_q}} (i\hat{e_q}.\vec{q}), \tag{15}$$

we obtain the following result, directly

$$\hat{H}_{el-ph} = \sum_q [c(q) a_q e^{i\vec{q}.\vec{r}} + c^*(q) a_q^\dagger e^{-i\vec{q}.\vec{r}}]. \tag{16}$$

The eigenstate of the phonon Hamiltonian in harmonic approximation is defined by $|n_q>$, where $n_q = \frac{1}{e^{(\frac{\hbar\omega}{k_B T})-1}}$ is the Bose-Einstein distribution function of phonon, so we can define a new basis as follows, $\mid \mathbf{k}\lambda n_q >=\mid \mathbf{k}\lambda > \otimes \mid n_q >$. Scattering matrix of electron-phonon interaction is given as follows

$$< \mathbf{k}'\lambda' n'_q \mid \hat{H}_{el-ph} \mid \mathbf{k}\lambda n_q > = \begin{cases} \delta_{n'_q, n_q - 1} \delta_{\lambda'\lambda} c(q) \sqrt{n_q}, & \text{if } \mathbf{k}' = \mathbf{k} + \mathbf{q}, \\ \\ \delta_{n'_q, n_q + 1} \delta_{\lambda'\lambda} c^*(q) \sqrt{n_q + 1}, & \text{if } \mathbf{k}' = \mathbf{k} - \mathbf{q}. \end{cases} \tag{17}$$

In this work we have employed the procedure which have been used in [18]. The two last terms of the Hamiltonian are responsible for both spin-dependent and spin-independent relaxation mechanisms. If we rename these two terms as

$$V = \hat{V}_{im} + \hat{H}_{el-ph} \tag{18}$$

The Lippman- Schwinger scattering state of a conduction electron reads,

$$\mid \mathbf{k}\lambda n_q >_{scat} \;=\; \mid \mathbf{k}\lambda n_q > + \sum_{\mathbf{k}'q'\lambda'} \frac{V_{\mathbf{k}'\lambda'\mathbf{n}'_\mathbf{q},\mathbf{k}\lambda\mathbf{n}_\mathbf{q}}}{\epsilon_{\mathbf{k}\lambda} - \epsilon_{\mathbf{k}'\lambda'} + i\eta} \mid \mathbf{k}'q'\lambda' >, \tag{19}$$

where η is a small positive quantity. Then the expectation value of the electron spin in a given scattering state is

$$S_{\lambda,i}(\mathbf{k}) = S^{(0)}_{\lambda,i}(\mathbf{k}) + \hbar \sum_{\mathbf{k}'q'\lambda'} Re[\frac{V_{k'\lambda'\acute{n}_q,\mathbf{k}\lambda n_q} < \sigma_i >_{\mathbf{k}'\lambda'\acute{n}_q,\mathbf{k}\lambda n_q}}{\epsilon_{\mathbf{k}\lambda} - \epsilon_{\mathbf{k}'\lambda'} + i\eta}]. \tag{20}$$

Here we have defined $S_{\lambda,i}(\mathbf{k}) =< \mathbf{k}\lambda n_q \mid \hat{S}_i \mid \mathbf{k}\lambda n_q >_{scat}$. Therefore one can obtain spin expectation value as seen in eq. (21).

$$\begin{aligned} S_{\lambda,i}(\mathbf{k}) \;&=\; S^{(0)}_{\lambda,i}(\mathbf{k}) + \hbar \sum_{\mathbf{k}'q'\lambda'} Re[V_{\mathbf{k}'\lambda'\acute{n}_q,\mathbf{k}\lambda n_q} < \hat{\sigma}_i >_{\mathbf{k}'\lambda'\acute{n}_q,\mathbf{k}\lambda n_q} Pr\frac{1}{\epsilon_{\mathbf{k}\lambda} - \epsilon_{\mathbf{k}'\lambda'}} \\ &- V_{k'\lambda'\acute{n}_q,\mathbf{k}\lambda n_q} < \sigma_i >_{\mathbf{k}'\lambda'\acute{n}_q,k\lambda n_q} i\pi\delta(\epsilon_{k\lambda} - \epsilon_{k'\lambda'})]. \end{aligned} \tag{21}$$

Here $< \sigma_i >_{\mathbf{k}'\lambda',\mathbf{k}\lambda}$ is the expectation value of the Pauli Matrix for a Lippman-Schwinger scattering state. Then, net spin density can be giving by

$$< S_i >= \sum_{\mathbf{k},q\lambda} S_{\lambda,i}(\mathbf{k}) f_\lambda(\mathbf{k},\mathbf{q}), \tag{22}$$

in which $f_\lambda(\mathbf{k},\mathbf{q})$ is the non-equilibrium distribution function of conduction electrons. In the absence of external electric field, this is reduced to the equilibrium Fermi-Dirac distribution,

$$f_\lambda(\mathbf{k},\mathbf{q}) = f_0(\epsilon_{\mathbf{k}\lambda}) = \frac{1}{1 + e^{\frac{(\epsilon_{\mathbf{k}\lambda} - \epsilon_F)}{k_B T}}}. \tag{23}$$

We have used the Debye model, so the summation over $\mathbf{q}$ is easily calculated by replacing it with an integral. This integral can be considered to be evaluated in the interval starting from $\mathbf{q} = 0$ to the Debye wave vector, $\mathbf{q}_D$. This wave vector is directly related to the free-electron Fermi wave vector. In two-dimensional metals, $q_D = \sqrt{\frac{2}{z}}k_F$, where k_F is the free-electron Fermi wave vector, and z is the nominal valence [26].

In the presence of scatterings, the non-equilibrium distribution function will be derived by solving the Boltzmann equation (in steady state for a homogeneous system),

$$\dot{\mathbf{k}}.\frac{\partial f_\lambda}{\partial \mathbf{k}} = (\frac{\partial f_\lambda}{\partial t})_{coll}, \tag{24}$$

where $\dot{\mathbf{k}} = \frac{-e\mathbf{E}}{\hbar}$ and $(\frac{\partial f_\lambda}{\partial t})_{coll}$ is called the collision integral due to scatterings, that in elastic scattering approximation reads [27]

$$\begin{aligned}(\frac{\partial f_\lambda}{\partial t})_{coll} &= -\sum_{\mathbf{k}'q'\lambda'} W_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q} f_\lambda(\mathbf{k},q)(1-f_{\lambda'}(\mathbf{k}',q'))\delta(\epsilon_{\mathbf{k}\lambda}-\epsilon_{\mathbf{k}'\lambda'}) \\ &+ \sum_{\mathbf{k}'q'\lambda'} W_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q} f_{\lambda'}(\mathbf{k}',q')(1-f_\lambda(\mathbf{k},q))\delta(\epsilon_{\mathbf{k}\lambda}-\epsilon_{\mathbf{k}'\lambda'}).\end{aligned} \tag{25}$$

$W_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q}$ are the transition probabilities that are given by the Fermi's golden rule, $W_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q} = \frac{2\pi}{\hbar}|V_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q}|^2$.

since $\delta_{n'_q,n_q}$ selects only the diagonal elements of $|V_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q}|^2$ while $\delta_{n'_q,n_q-1}\delta_{\mathbf{k}',\mathbf{k}+\mathbf{q}}$ and $\delta_{n'_q,n_q+1}\delta_{\mathbf{k}',\mathbf{k}-\mathbf{q}}$ select some of the non-diagonal elements of $|V_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q}|^2$, therefore one can easily obtain

$$|V_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q}|^2 = |<\mathbf{k}'\lambda' n'_q \mid \hat{H}_{el-ph} \mid \mathbf{k}\lambda n_q>|^2 + |<\mathbf{k}'\lambda' n'_q \mid V_{im} \mid \mathbf{k}\lambda n_q>|^2, \tag{26}$$

and accordingly

$$W_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q} = W^{(1)}_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q} + W^{(2)}_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q} + W^{(3)}_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q}. \tag{27}$$

$W^{(1)}_{\mathbf{k'}\lambda' n\acute{q},\mathbf{k}\lambda n_q}$ is due to impurity potential,

$$W^{(1)}_{\mathbf{k'}\lambda' n\acute{q},\mathbf{k}\lambda n_q} = \begin{pmatrix} 0 & J^2 \\ J^2 & 0 \end{pmatrix} n_i \delta_{n'_q,n_q}. \tag{28}$$

In which we have used the following approximation, $(1/L_x L_y)\sum_j \sum_{j'} \exp(i(\vec{k'} - \vec{k}).(\vec{r}_j - \vec{r}_{j'})) = n_i$, where n_i is the impurity density, L_x, L_y are system dimensions and it should be noted that the summations have been performed over the random positions of impurities.

$W^{(2)}_{\mathbf{k'}\lambda' n\acute{q},\mathbf{k}\lambda n_q}$ and $W^{(3)}_{\mathbf{k'}\lambda' n\acute{q},\mathbf{k}\lambda n_q}$ are due to electron-phonon interaction, where $W^{(2)}_{\mathbf{k'}\lambda' n\acute{q},\mathbf{k}\lambda n_q}$ is for phonon absorbtion contribution,

$$W^{(2)}_{\mathbf{k'}\lambda' n\acute{q},\mathbf{k}\lambda n_q} = \delta_{n'_q,n_q-1}\delta_{\mathbf{k'},\mathbf{k}+\mathbf{q}}\delta_{\lambda',\lambda} \begin{pmatrix} c_q\sqrt{n_q} & 0 \\ 0 & c_q\sqrt{n_q} \end{pmatrix}, \tag{29}$$

and $W^{(3)}_{\mathbf{k'}\lambda' n\acute{q},\mathbf{k}\lambda n_q}$ should be considered for the case of emission,

$$W^{(3)}_{\mathbf{k'}\lambda' n\acute{q},\mathbf{k}\lambda n_q} = \delta_{n'_q,n_q+1}\delta_{\mathbf{k'},\mathbf{k}-\mathbf{q}}\delta_{\lambda',\lambda} \begin{pmatrix} c^*_q\sqrt{n_q+1} & 0 \\ 0 & c^*_q\sqrt{n_q+1} \end{pmatrix}. \tag{30}$$

Since the energy dispersion of conduction electrons becomes anisotropic in the presence of Rashba and Dresselhaus spin-orbit interactions, this manifest itself in the scattering process and one can choose an anisotropic solution to the Boltzmann equation as follows [18],

$$\delta f_\lambda(\mathbf{k},\mathbf{q}) = \mathbf{e}\frac{\partial \mathbf{f_0}(\epsilon_{\mathbf{k}\lambda})}{\partial \epsilon_{\mathbf{k}\lambda}}[\mathbf{a}_{\mathbf{kq}\lambda}(\mathbf{E}.\ \mathbf{v}_{\mathbf{k}\lambda}) + \mathbf{b}_{\mathbf{kq}\lambda}(\mathbf{E}\times\hat{\mathbf{e}}_\mathbf{z}).\mathbf{v}_{\mathbf{k}\lambda}]. \tag{31}$$

$$\mathbf{v}_{\mathbf{k}\lambda} = \frac{1}{\hbar}\nabla_\mathbf{k}\epsilon_{\mathbf{k}\lambda}, \tag{32}$$

where $\mathbf{v}_{\mathbf{k}\lambda}$ is the velocity of conduction electrons, $\delta f_\lambda = f_\lambda - f_0$, $\hat{e}_z$ is a unit vector along the direction of normal of the two-dimensional plane, $a_{\mathbf{k}q\lambda}$

and $b_{\mathbf{k}q\lambda}$ are two unknown coefficients that can determine self-consistently by using eq. (24) and eq. (31) in which $b_{\mathbf{k}q\lambda}$ arise due to the anisotropic nature of the system. Then one can find that the unknown coefficients $a_{\mathbf{k}q\lambda}$ and $b_{\mathbf{k}q\lambda}$ are satisfying the following equations [18]

$$\frac{a_{\mathbf{k}q\lambda}}{\tau^{(1)}_{\mathbf{k}q\lambda}} + \frac{b_{\mathbf{k}q\lambda}}{\tau^{(2)}_{\mathbf{k}q\lambda}} = 1, \tag{33}$$

$$\frac{a_{\mathbf{k}q\lambda}}{\tau^{(2)}_{\mathbf{k}q\lambda}} - \frac{b_{\mathbf{k}q\lambda}}{\tau^{(1)}_{\mathbf{k}q\lambda}} = 0, \tag{34}$$

From Eqs. (33) and (34), one easily gets

$$a_{\mathbf{k}q\lambda} = \frac{\tau^{(1)}_{\mathbf{k}q\lambda}}{1 + [\frac{\tau^{(1)}_{\mathbf{k}q\lambda}}{\tau^{(2)}_{\mathbf{k}q\lambda}}]^2}, \qquad b_{\mathbf{k}q\lambda} = \frac{\tau^{(2)}_{\mathbf{k}q\lambda}}{1 + [\frac{\tau^{(2)}_{\mathbf{k}\lambda}}{\tau^{(1)}_{\mathbf{k}q\lambda}}]^2}, \tag{35}$$

in which $\tau^{(1)}_{\mathbf{k}q\lambda}$ and $\tau^{(2)}_{\mathbf{k}q\lambda}$ are two relaxation times, and are defined by

$$\frac{1}{\tau^{(1)}_{\mathbf{k}q\lambda}} = \sum_{\mathbf{k}'q',\lambda'} W_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q} \{1 - \frac{|\mathbf{v}_{\mathbf{k}'\lambda'}|}{|\mathbf{v}_{\mathbf{k}'\lambda'}|} \cos[\theta(\mathbf{v}_{\mathbf{k}\lambda} \wedge \mathbf{v}_{\mathbf{k}'\lambda'})]\}, \tag{36}$$

$$\frac{1}{\tau^{(2)}_{\mathbf{k}q\lambda}} = \sum_{\mathbf{k}'q',\lambda'} W_{\mathbf{k}'\lambda' n\acute{q},\mathbf{k}\lambda n_q} \frac{|\mathbf{v}_{\mathbf{k}'\lambda'}|}{|\mathbf{v}_{\mathbf{k}'\lambda'}|} \sin[\theta(\mathbf{v}_{\mathbf{k}\lambda} \wedge \mathbf{v}_{\mathbf{k}'\lambda'})], \tag{37}$$

where $\theta(\mathbf{v}_{\mathbf{k}\lambda} \wedge \mathbf{v}_{\mathbf{k}'\lambda'})$ is the angle between $\mathbf{v}_{\mathbf{k}\lambda}$ and $\mathbf{v}_{\mathbf{k}'\lambda'}$. The spin current operator is defined as [20]

$$\hat{J}^i_x = \frac{\hbar}{2}\{\sigma_i, \hat{v}_x\}, \tag{38}$$

where $\hat{v}_x = \hbar^{-1}(\frac{\partial \hat{H}}{\partial k_x})$ is the velocity operator. The expectation value of the spin current in a given scattering state i.e. $J^i_x(\mathbf{k},\lambda) =< \mathbf{k}\lambda n_q \mid \hat{J}^i_x \mid \mathbf{k}\lambda n_q >_{scat}$

can be obtained as shown in eq. (39).

$$
\begin{aligned}
J_x^i(\mathbf{k},\lambda) \quad = \quad & J_x^{i(0)}(\mathbf{k},\lambda) + \hbar \sum_{\mathbf{k}'q'\lambda'} Re[V_{\mathbf{k}'\lambda'\acute{n}_q,\mathbf{k}\lambda n_q} < J_x^i >_{\mathbf{k}'\lambda'\acute{n}_q,\mathbf{k}\lambda n_q} Pr\frac{1}{\epsilon_{\mathbf{k}\lambda} - \epsilon_{\mathbf{k}'\lambda'}} \\
& -V_{\mathbf{k}'\lambda'\acute{n}_q,\mathbf{k}\lambda n_q} < J_x^i >_{\mathbf{k}'\lambda'\acute{n}_q,k\lambda n_q} i\pi\delta(\epsilon_{k\lambda} - \epsilon_{k'\lambda'})]. \qquad (39)
\end{aligned}
$$

In which we have defined $J_x^{i(0)} =< \mathbf{k}\lambda n_q \mid \hat{J}_x^i \mid \mathbf{k}\lambda n_q >$.

Then the transport spin current in x direction with spin parallel to the x and y axes, is given by

$$
J_x^i = \sum_{\mathbf{k}q,\lambda} \hat{J}_x^i(\mathbf{k},\lambda)\delta f_\lambda(\mathbf{k},\mathbf{q}), \qquad (i = x, y \quad or \quad s_x, s_y) \qquad (40)
$$

3. Results

In the current work the spin accumulation and spin-current of a two dimensional electron gas system have been obtained in the presence of the Rashba, Dreesselhaus and electron-phonon interactions. This has been accomplished by utilizing a semi-classical model developed for anisotropic systems. This anisotropy can be induced by spin-orbit couplings in scattering matrix or in the energy dispersion. As mentioned before the Rashba and Dreesselhaus couplings can not be responsible for spin-current generation in non-equilibrium regime [18, 22]. Meanwhile non-equilibrium spin accumulation can be effectively controlled by these spin-orbit interactions [18]. In the present work it was shown that the electron-phonon interaction has a considerable role in the generation of spin-current which was expected to be obtained by spin-orbit couplings. The spin accumulation of the system has also been controlled by the strength of the electro-phonon coupling.

In the present system electric field is assumed to be applied along the x direction and the numerical parameters have been chosen as follows $\epsilon_f = 10eV$ is the Fermi energy, $J = 0.1eV$, $n_i = 10^{10}cm^{-2}$ is the density of impurities, $T = 1K$ and $V_s = 4950m/s$. In addition Rashba and Dreesselhaus couplings have been denoted by $\epsilon_\alpha = m\alpha^2/\hbar^2$,$\epsilon_\beta = m\beta^2/\hbar^2$ respectively. The Rashba coupling can reach high values up to 0.2 eV for example in epitaxial graphene grown on a Ni(111) substrate [28]. However in the present work a typically lower range has been chosen for SOC as reported for other materials.

It was found out that the electron-phonon interaction can not be considered as a underestimate effect on spin-dependent mechanisms. It was demonstrated that at low electron-phonon coupling strengths the lattice vibrations are more effective.

In Fig. 5 and Fig. 5 longitudinal spin-current has been depicted as a function of the deformation potential. Spin-current has been induced due to the lattice-electron interactions. These figures clearly show that spin-current of the system has a accountable value in which its sign and magnitude can be controlled by the SOCs. At the same time, as shown in Figs. 5 and 5 longitudinal and transverse spin accumulations can be effectively changed by the Rashba and Dreesselhaus couplings.

Therefore forgoing results show that when the effect of electron-phonon interaction is taken into account, in the semi-classical regime it turns out that the both components of the spin-current can take non-zero values. It seems that the details of the scattering potential has an important role in the the generation of the spin current in the presence of the Rashba and Dreesselhaus couplings. As reported in [18] Huang and Hu found that short-range

delta function impurity scatterings (where actually have spherical symmetry) result in zero spin-polarized current in the system. As mentioned before anisotropic effects can be induced by two different sources: the energy dispersion relation and the scattering matrix of the relaxations. The first source of the anisotropic is small and can be neglected. Meanwhile the anisotropic effects of the scattering matrix can be regarded as anisotropy of the eigen-states and the anisotropy of the scattering potential. For isotropic scattering potentials the anisotropy of the response functions are just given by the eigen-states. Therefore redistribution of the carriers' population by the anisotropic or inelastic relaxation mechanisms can change the ensemble average of Rashba and Dreesselhaus k-dependent effective field. This can produce non-vanishing spin current in the system since the effective field in the present case has been modified by the in elastic scatterings. Increasing the electron-phonon coupling strength decreases both spin current and spin accumulation of the system as shown in Fig. 5- Fig. 5. Unlike the intermediate range of the deformation potential at high electron-phonon couplings the momentum of the electrons has been effectively randomized by the electron-phonon interaction since the relaxation time of the states decreases. Therefore in this case, the population of the carriers approaches to the limit of the isotropic scatterings in which the spin current of the system vanishes.

An important feature which can be inferred from the results is the fact that, the absolute value of spin-current and spin accumulation decreases for high electron-phonon couplings as depicted in Fig. 5 and Fig. 5. Spin-current induced by the lattice longitudinal vibrations disappears in the limit of high deformation potential and rapidly increases for low electron-phonon couplings.

Unlike the spin-orbit couplings, electron-phonon interaction can change the order of magnitude of the spin-current. However it should be noted that numerical results reveals that, in the limit of $D_{ac} \to 0$ spin current vanishes abruptly. and it was a numerical discontinuity (has not included in the figures) at $D_{ac} = 0$. Numerical results show that for $D_{ac} = 0$ and in the case of nonmagnetic impurities, $J = 0$ spin-current identically vanishes. This is in agreement with the results that have been pointed out in Ref. [18] for identical conditions.

4. Conclusion

In this work a semi-classical approach have been implemented for the study of the magnetoelectric effects of a 2DEG system. The primary focus of this work is to show that the electron-phonon coupling has an important role for generation of the spin-current in non-equilibrium regime. Since it was verified that the Rashba and Dreesselhaus couplings (when the electron-phonon coupling is absent) can not be responsible for spin-current in this regime. It was numerically verified that even at low electron-phonon couplings the lattice vibrations can mediate in the spin-transport process modulated by spin-orbit interactions.

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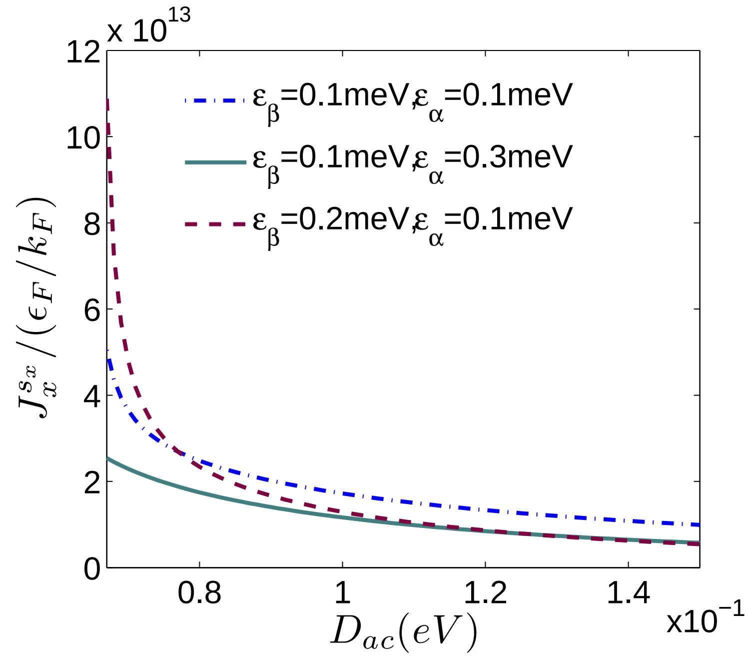
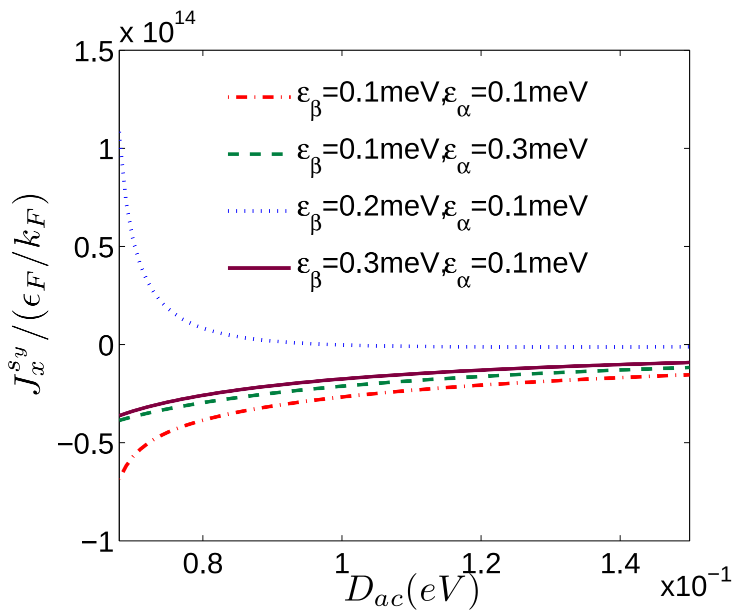
Fig. 1: Longitudinal spin current as a function of the deformation potential for different SO couplings.

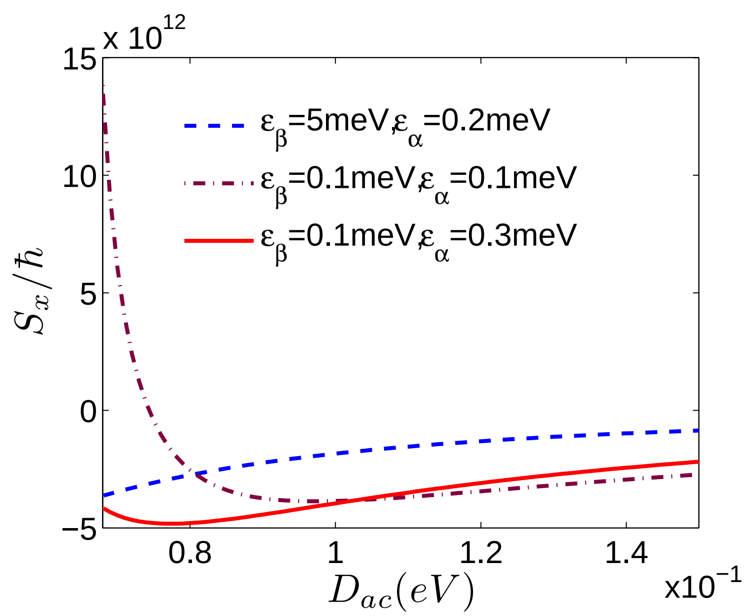
Fig. 2: Transverse spin current as a function of the deformation potential for different SO couplings.

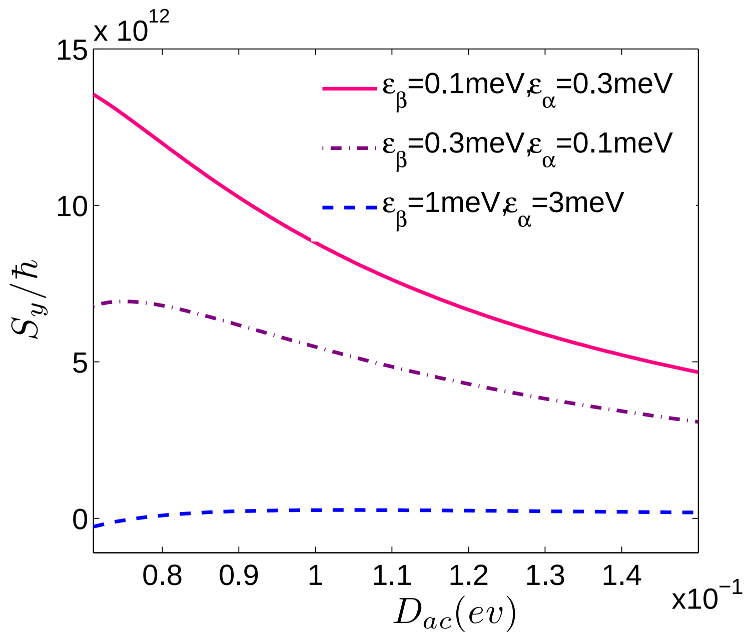
Fig. 3: Longitudinal spin accumulation as a function of the deformation potential for different SO couplings.

Fig. 4: Transverse spin accumulation as a function of the deformation potential for different SO couplings.


x 10^13
12
10
8
6
4
2
0
$J_x^{s_x}/(\epsilon_F/k_F)$
ε_β=0.1meV,ε_α=0.1meV
ε_β=0.1meV,ε_α=0.3meV
ε_β=0.2meV,ε_α=0.1meV
0.8
1
1.2
1.4
x10^-1
$D_{ac}(eV)$


x 10^14
ε_β=0.1meV,ε_α=0.1meV
ε_β=0.1meV,ε_α=0.3meV
ε_β=0.2meV,ε_α=0.1meV
ε_β=0.3meV,ε_α=0.1meV
$J_x^{s_y}/(\epsilon_F/k_F)$
$D_{ac}(eV)$
x10^−1


x 10^12
ε_β=5meV,ε_α=0.2meV
ε_β=0.1meV,ε_α=0.1meV
ε_β=0.1meV,ε_α=0.3meV
$S_x/\hbar$
$D_{ac}(eV)$
x10^−1



x 10^12
15
10
5
0
$S_y/\hbar$
0.8
0.9
1
1.1
1.2
1.3
1.4
x10^−1
$D_{ac}(ev)$
ε_β=0.1meV,ε_α=0.3meV
ε_β=0.3meV,ε_α=0.1meV
ε_β=1meV,ε_α=3meV